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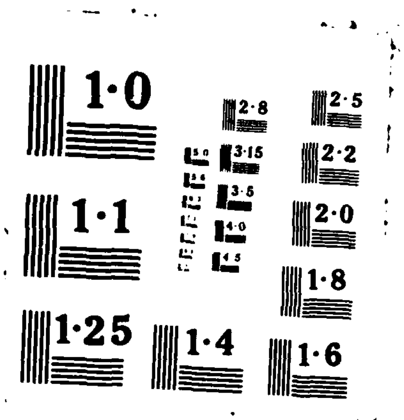
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WAVE CURVES FOR SIMPLE ELASTIC SOLIDS

FINAL REPORT

by

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November 11, 1987

U. S. ARMY RESEARCH OFFICE

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Simple waves; Shock waves; Wave curves, Riemann problem, Hyperbolic systems.

19 ABSTRACT (Continue on reverse if necessary and identify by block number)

This report summarizes the results of the research project supported by the Army Research Office. The project was to study the wave curves for one-dimensional Riemann problem which arises in the problem of plane waves in isotropic elastic materials. The concept of wave curves is to be extended to the two-dimensional Riemann problem which arises in the problem of oblique shocks impinging on the boundary. Satisfactory results are obtained and are reported here.

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I. STATEMENT OF THE PROBLEM STUDIED

The research project supported by the Army Research Office was to study wave curves in simple isotropic elastic solids. Wave curves consist of simple wave curves and shock wave curves. They are essential in the solution of the Riemann problem which can, in turn, be used to build the solution for more general initial and boundary value problems. The project was to study the one-dimensional Riemann problem first. Following that, the two-dimensional Riemann problem which arises in the problem of an oblique plane shock impinging on a plane boundary is to be studied.

II. SUMMARY OF THE RESULTS OBTAINED

For the one-dimensional Riemann problem we consider plane waves in a hyperelastic half-space. The governing differential equations are a 6 x 6 system of hyperbolic conservation laws. There are six wave speeds $\mp c_i$, $i = 1,2,3$, of which the c_2 - characteristic field is linearly degenerate. Since only the positive wave speeds are relevant for the modified Riemann problem in which the initial and boundary conditions are constant, one need consider only the c_1 and c_3 characteristic fields. Therefore the system is reduced to a two-wave speed system.

For the second-order hyperelastic materials in which the complimentary strain energy can be expanded in terms of stress components of up to the third-order in the stress, it is shown that there exists an umbilic point at which $c_1 = c_3$. Thus the system is not strictly hyperbolic. Wave curves are presented for a fixed initial condition with arbitrary boundary conditions. For a different choice of the initial condition, we would obtain a different geometry for the wave curves. Even though there are four material constants, it is shown that the wave curves depend on one non-dimensional parameter k. The wave curves for all possible choice of k and all possible combinations of initial and boundary conditions are presented in [1] which contains 86 pages with 45 figures. Due to the non-strictly hyperbolic nature of the system, several unexpected and interesting phenomena are discovered. For instance, the wave curve may have a terminal point. The wave curve which starts from a given point may intersect with the other wave curve which starts from the same point. Also, the shock wave with shock wave speed V_i, $i = 1$ or 3, may satisfy Lax stability conditions for both $i = 1$ and 3 or may not satisfy Lax stability condition for either $i = 1$ or 3. These and other abnormal results can be found in [1].

Application of the solution to the one-dimensional Riemann problem is given in [4] where the problem of impact on a thin-walled tube is considered. The mathematical formulation and the wave curves are almost identical to that of plane waves in a half-space except that the c_2 characteristic field is absent here.

We also considered one-dimensional Riemann problems for inelastic materials. In [3] we studied the waves in an elastic-plastic material and focused our attention on the discontinuities across an elastic-plastic boundary.

On the two-dimensional Riemann problem, the concept of simple wave curves and shock wave curves still applies. However, for the one-dimensional Riemann problem the wave curves are typically curves in two- or three-dimensional space. For the two-dimensional Riemann problem the wave curves are curves in four or higher dimensional space. To simplify the analysis we consider, as a beginning, waves in incompressible hyperelastic materials. With the assumption of incompressibility there are two wave speeds instead of three. Therefore one need consider only two wave fans instead of three. The two-dimensional Riemann problem arises in the problem of oblique shocks impinging on a boundary. The solution of the Riemann problem provides the reflected waves due to the incident shock. This is presented in [5].

A problem related to [5] is the one in which the incident shock wave becomes a grazing incident wave. If the boundary is a free-surface, we have a surface wave in the half-space. The solution to the surface waves in an anisotropic elastic half-space hinged on three tensors known as the Barnett-Lothe tensors. The properties of these tensors are crucial in determining the surface wave speed. We found some interesting properties of these tensors in [2].

III. PUBLICATIONS UNDER THIS PROJECT

[1] Zhijing Tang and T. C. T. Ting, "Wave Curves for the Riemann Problem of Plane Waves in Isotropic Elastic Solids," Int. J. Engineering Science. In press.

[2] P. Chadwick and T. C. T. Ting, "On the Structure and Invariance of the Barnett-Lothe Tensors," Q. Appl. Math. In press.

[3] T. C. T. Ting, "Impossibility of Higher Order Discontinuities Across an Elastic-Plastic Boundary in Elastic-Plastic Wave Propagation," The Robotnov Memorial Volume on Dynamic Plasticity. In press.

[4] T. C. T. Ting, Zhijing Tang and Yongchi Li, "Impact on Nonlinear Elastic Thin-Walled Tube," Proc. 9th SMIRT Post-Conference Seminar on Impact. In press.

[5] Zhijing Tang and T. C. T. Ting, "Simple Waves and Shock Waves in Two-Dimensional Incompressible Hyperelastic Solids," under preparation.

IV. SCIENTIFIC PERSONNEL PARTICIPATED IN THE PROJECT

Nihal Somaratna
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END

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